B-field of a horizontal bar magnet

$C := 1$ coefficient to linearly scale everything as needed.

$a := 250 \quad b := 50 \quad R := \frac{b}{2}$

a = length of bar magnet in arbritrary units b=width of bar magnet

R = radius of bar magnet

The following are elliptic integrals for x and y components of the B-field. Magnet is oriented horizontally along the x-direction. These integrals have no analytical solution. The field is scaled so the B at the center of an infinitely long bar magnet is 1. The field at the center of a finite-length magnet is a little less than 1.

$$Bx(x,y) := \frac{C \cdot R}{4 \cdot \pi} \cdot \int_{\frac{-a}{2}}^{\frac{a}{2}} \int_{0}^{2 \cdot \pi} \frac{(R - y \cdot \sin(phi))}{\left[R^2 + y^2 - 2 \cdot y \cdot R \cdot \sin(phi) + (x - xp)^2\right]^{\frac{3}{2}}} \, dphi \, dxp$$

$$By(x,y) := \frac{C \cdot R}{4 \cdot \pi} \cdot \int_{\frac{-a}{2}}^{\frac{a}{2}} \int_{0}^{2 \cdot \pi} \frac{(x - xp) \cdot \sin(phi)}{\left[R^2 + y^2 - 2 \cdot y \cdot R \cdot \sin(phi) + (x - xp)^2\right]^{\frac{3}{2}}} \, dphi \, dxp$$

Check: B at middle of right edge of magnet should be about 1/2 of B-field at center.

$Bx(0,0) = 0.980581 \qquad Bx\left(\frac{a}{2},0\right) = 0.4975 \qquad \frac{Bx\left(\frac{a}{2},0\right)}{Bx(0,0)} = 0.50737$

$B\_mag(x,y) := \sqrt{Bx(x,y)^2 + By(x,y)^2}$ Magnitude of B-field (unused)

$$B(y,z) := \begin{pmatrix} Bx(y,z) \\ By(y,z) \end{pmatrix}$$ Actual B-field

Following line scales B so that it does not appear to fall off with distance too rapidly.
THIS ASSUMES THAT B=1 at CENTER OF MAGNET. (This is unused in this mathcad file).

$n := 4$ n is a number between 1(no rescaling) and, say, 5 (big rescaling)

$$B\_dir(x,y) := \begin{bmatrix} \dfrac{Bx(x,y)}{B\_mag(x,y)} \\ \left(\dfrac{By(x,y)}{B\_mag(x,y)}\right) \end{bmatrix} \qquad Bscaled(x,y) := \begin{bmatrix} \sqrt[n]{B\_mag(x,y)} \cdot \dfrac{Bx(x,y)}{B\_mag(x,y)} \\ \left(\sqrt[n]{B\_mag(x,y)} \cdot \dfrac{By(x,y)}{B\_mag(x,y)}\right) \end{bmatrix}$$

Compute field interior to magnet:

$b = 50 \qquad a = 250$

$xrange := \dfrac{a}{2} \qquad yrange := \dfrac{b}{2} \qquad xdel := 5 \qquad ydel := 5$

$i := 0 .. \dfrac{xrange}{xdel} \qquad j := 0 .. \dfrac{yrange}{ydel}$

$gridOffset := 0.1$ Offset is so that none of the points fall exactly on the edge of the magnet, where the integrals do not converge.

$xind_i := i \cdot xdel - gridOffset \qquad yind_j := j \cdot ydel - gridOffset$

$M_{i,j} := B\left(xind_i, yind_j\right)$

$iMax := \dfrac{xrange}{xdel} \qquad jMax := \dfrac{yrange}{ydel}$

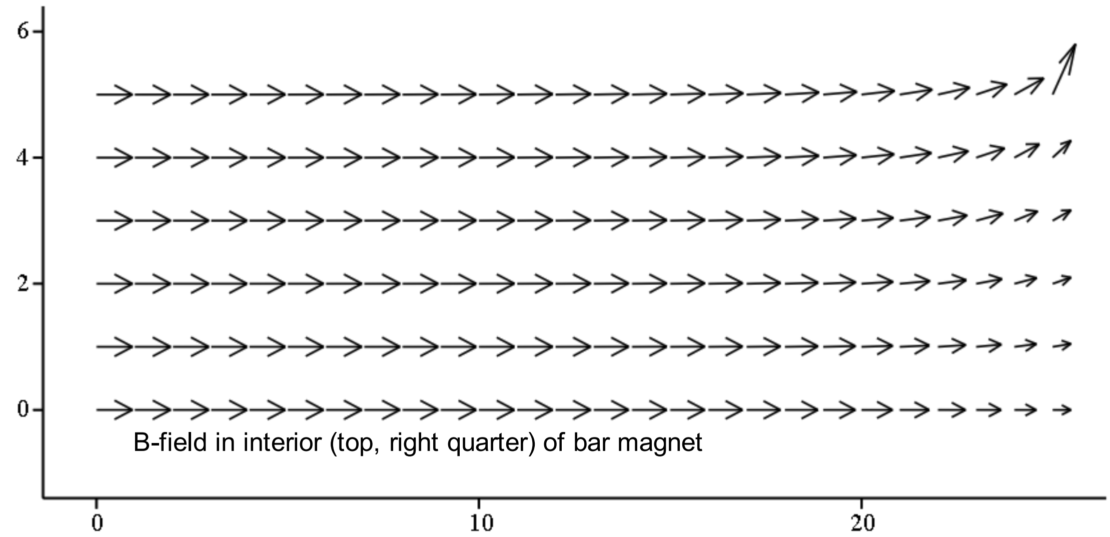
$BX\_int_{i,j} := \left(M_{i,j}\right)_0$

$BY\_int_{i,j} := \left(M_{i,j}\right)_1$

$iMax = 25 \qquad jMax = 5$

$BXrounded\_int_{i,j} := round\left(BX\_int_{i,j}, 6\right)$

$BYrounded\_int_{i,j} := round\left(BY\_int_{i,j}, 6\right)$



(BX_int, BY_int)

Compute field outside magnet, nearby with dense grid

$xrange := 2a \qquad yrange := 8 \cdot b \qquad xdel := 5 \qquad ydel := 5$

$i := 0 .. \frac{xrange}{xdel} \qquad j := 0 .. \frac{yrange}{ydel} \qquad xrange = 500$

$iMax := \frac{xrange}{xdel} \quad jMax := \frac{yrange}{ydel} \qquad yrange = 400$

$iMax = 100 \qquad jMax = 80$

$gridOffset := 0.1$ Offset is so that none of the points fall exactly on the edge of the magnet, where the integrals do not converge.

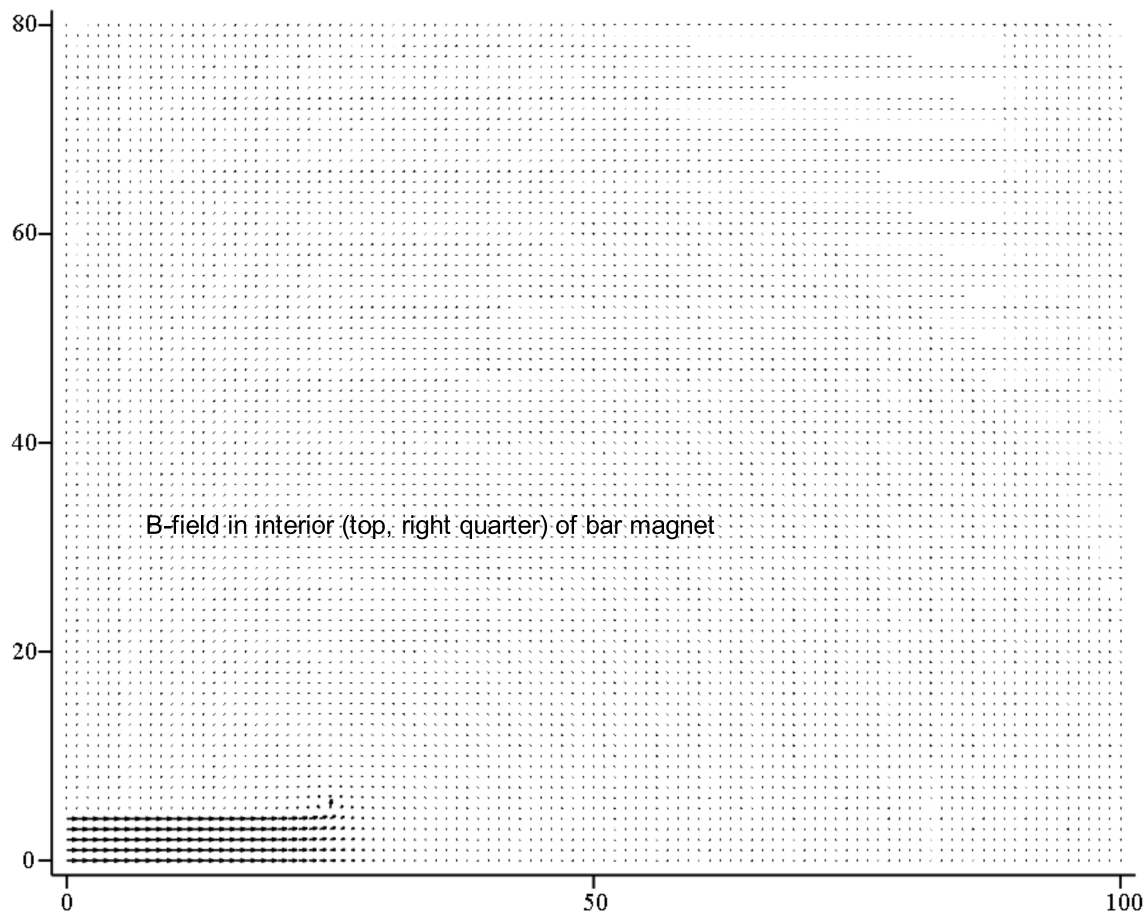
$xind_i := i \cdot xdel + gridOffset \qquad yind_j := j \cdot ydel + gridOffset$

$M_{i,j} := B\left(xind_i, yind_j\right)$

$BX\_ext1_{i,j} := \left(M_{i,j}\right)_0 \qquad BY\_ext1_{i,j} := \left(M_{i,j}\right)_1$

$BXrounded\_ext1_{i,j} := round(BX\_ext1_{i,j}, 6)$

$BYrounded\_ext1_{i,j} := round(BY\_ext1_{i,j}, 6)$



(BX_ext1, BY_ext1)

Compute B-field far away, with sparse grid.

$xrange := 10 \cdot a$ $yrange := 24 \cdot b$ $xdel := 20$ $ydel := 20$

$i := 0 .. \frac{xrange}{xdel}$ $j := 0 .. \frac{yrange}{ydel}$ $xrange = 2.5 \times 10^3$

$yrange = 1.2 \times 10^3$

$gridOffset := 0$ Offset is so that none of the points fall exactly on the edge of the magnet, where the integrals do not converge.

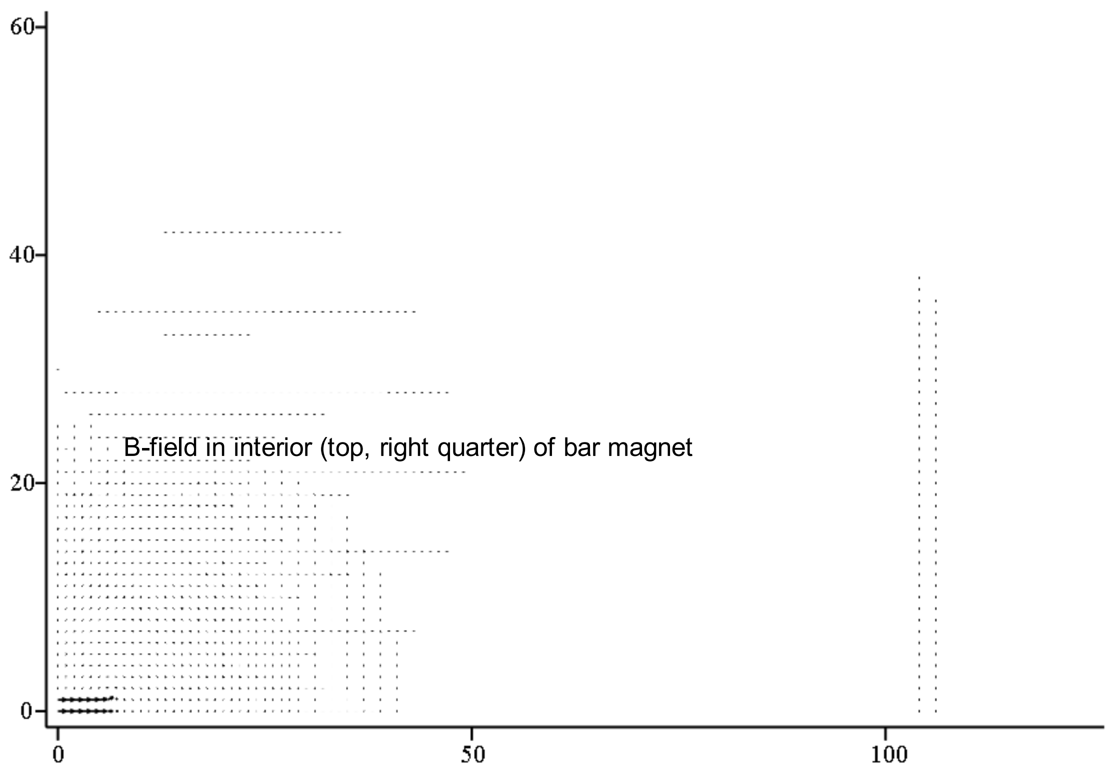
$xind_i := i \cdot xdel + gridOffset \qquad yind_j := j \cdot ydel + gridOffset$

$M_{i,j} := B\left(xind_i, yind_j\right)$

$BX\_ext2_{i,j} := \left(M_{i,j}\right)_0 \qquad BY\_ext2_{i,j} := \left(M_{i,j}\right)_1$

$BXrounded\_ext2_{i,j} := round\left(BX\_ext2_{i,j}, 7\right)$

$BYrounded\_ext2_{i,j} := round\left(BY\_ext2_{i,j}, 7\right)$



(BX_ext2 , BY_ext2)

BXrounded_int =

	0	1	2	3	4	5
10	0.969	0.97	0.97	0.971	0.972	0.97
11	0.966	0.966	0.967	0.968	0.969	0.968
12	0.962	0.962	0.963	0.964	0.966	0.965
13	0.957	0.958	0.959	0.96	0.962	0.961
14	0.951	0.952	0.953	0.955	0.957	0.957
15	0.944	0.944	0.946	0.948	0.951	0.952
16	0.934	0.934	0.936	0.94	0.944	0.946
17	0.921	0.922	0.925	0.929	0.935	0.939
18	0.904	0.905	0.909	0.915	0.923	0.93

19	0.881	0.883	0.889	0.898	0.909	0.918
20	0.851	0.854	0.861	0.874	0.89	0.905
21	0.81	0.814	0.824	0.841	0.864	0.889
22	0.756	0.76	0.773	0.796	0.829	0.866
23	0.685	0.689	0.702	0.729	0.776	0.839
24	0.597	0.6	0.61	0.632	0.683	0.801
25	0.5	0.5	0.5	0.5	0.502	...

BYrounded_int =

	76	77	78	7

BXrounded_ext1 =

0	$-6.12\cdot10^{-4}$	$-5.91\cdot10^{-4}$	$-5.7\cdot10^{-4}$	-5.5
1	$-6.12\cdot10^{-4}$	$-5.9\cdot10^{-4}$	$-5.7\cdot10^{-4}$	-5
2	$-6.11\cdot10^{-4}$	$-5.89\cdot10^{-4}$	$-5.69\cdot10^{-4}$	-5
3	$-6.09\cdot10^{-4}$	$-5.88\cdot10^{-4}$	$-5.67\cdot10^{-4}$	-5.4
4	$-6.06\cdot10^{-4}$	$-5.85\cdot10^{-4}$	$-5.65\cdot10^{-4}$	-5.4
5	$-6.03\cdot10^{-4}$	$-5.82\cdot10^{-4}$	$-5.62\cdot10^{-4}$	-5.4
6	$-5.99\cdot10^{-4}$	$-5.78\cdot10^{-4}$	$-5.59\cdot10^{-4}$	-5
7	$-5.94\cdot10^{-4}$	$-5.74\cdot10^{-4}$	$-5.54\cdot10^{-4}$	-5.3
8	$-5.89\cdot10^{-4}$	$-5.69\cdot10^{-4}$	$-5.5\cdot10^{-4}$	-5.3
9	$-5.83\cdot10^{-4}$	$-5.63\cdot10^{-4}$	$-5.44\cdot10^{-4}$	-5.2
10	$-5.77\cdot10^{-4}$	$-5.57\cdot10^{-4}$	$-5.38\cdot10^{-4}$	-5.2
11	$-5.69\cdot10^{-4}$	$-5.5\cdot10^{-4}$	$-5.32\cdot10^{-4}$	-5.1
12	$-5.61\cdot10^{-4}$	$-5.43\cdot10^{-4}$	$-5.25\cdot10^{-4}$	-5.0
13	$-5.53\cdot10^{-4}$	$-5.35\cdot10^{-4}$	$-5.17\cdot10^{-4}$	-5.0
14	$-5.44\cdot10^{-4}$	$-5.26\cdot10^{-4}$	$-5.09\cdot10^{-4}$	-4.9
15	$-5.34\cdot10^{-4}$	$-5.17\cdot10^{-4}$	$-5.01\cdot10^{-4}$	-4.8

BYrounded_ext1 =

	0	1	2	3
1	$2\cdot10^{-6}$	$9.1\cdot10^{-5}$	$1.78\cdot10^{-4}$	$2.61\cdot10^{-4}$
2	$4\cdot10^{-6}$	$1.82\cdot10^{-4}$	$3.57\cdot10^{-4}$	$5.24\cdot10^{-4}$
3	$5\cdot10^{-6}$	$2.79\cdot10^{-4}$	$5.46\cdot10^{-4}$	$8.02\cdot10^{-4}$
4	$8\cdot10^{-6}$	$3.84\cdot10^{-4}$	$7.52\cdot10^{-4}$	$1.103\cdot10^{-3}$
5	$1\cdot10^{-5}$	$5.01\cdot10^{-4}$	$9.81\cdot10^{-4}$	$1.437\cdot10^{-3}$
6	$1.3\cdot10^{-5}$	$6.35\cdot10^{-4}$	$1.241\cdot10^{-3}$	$1.816\cdot10^{-3}$
7	$1.6\cdot10^{-5}$	$7.9\cdot10^{-4}$	$1.543\cdot10^{-3}$	$2.255\cdot10^{-3}$
8	$1.9\cdot10^{-5}$	$9.73\cdot10^{-4}$	$1.9\cdot10^{-3}$	$2.773\cdot10^{-3}$
9	$2.4\cdot10^{-5}$	$1.195\cdot10^{-3}$	$2.329\cdot10^{-3}$	$3.393\cdot10^{-3}$
10	$2.9\cdot10^{-5}$	$1.466\cdot10^{-3}$	$2.854\cdot10^{-3}$	$4.148\cdot10^{-3}$
11	$3.6\cdot10^{-5}$	$1.804\cdot10^{-3}$	$3.506\cdot10^{-3}$	$5.083\cdot10^{-3}$
12	$4.4\cdot10^{-5}$	$2.231\cdot10^{-3}$	$4.328\cdot10^{-3}$	$6.256\cdot10^{-3}$
13	$5.5\cdot10^{-5}$	$2.779\cdot10^{-3}$	$5.381\cdot10^{-3}$	$7.751\cdot10^{-3}$
14	$6.9\cdot10^{-5}$	$3.495\cdot10^{-3}$	$6.75\cdot10^{-3}$	$9.684\cdot10^{-3}$
15	$8.8\cdot10^{-5}$	$4.444\cdot10^{-3}$	$8.559\cdot10^{-3}$	0.012
16	$1.13\cdot10^{-4}$	$5.72\cdot10^{-3}$	0.011	0.016

BXrounded_ext2 =

	56	57	58	59	
0	$-2.73\cdot10^{-5}$	$-2.59\cdot10^{-5}$	$-2.46\cdot10^{-5}$	$-2.34\cdot10^{-5}$	-
1	$-2.73\cdot10^{-5}$	$-2.59\cdot10^{-5}$	$-2.46\cdot10^{-5}$	$-2.34\cdot10^{-5}$	-
2	$-2.72\cdot10^{-5}$	$-2.58\cdot10^{-5}$	$-2.45\cdot10^{-5}$	$-2.33\cdot10^{-5}$	-
3	$-2.7\cdot10^{-5}$	$-2.56\cdot10^{-5}$	$-2.43\cdot10^{-5}$	$-2.31\cdot10^{-5}$	
4	$-2.67\cdot10^{-5}$	$-2.54\cdot10^{-5}$	$-2.41\cdot10^{-5}$	$-2.29\cdot10^{-5}$	-
5	$-2.64\cdot10^{-5}$	$-2.51\cdot10^{-5}$	$-2.38\cdot10^{-5}$	$-2.27\cdot10^{-5}$	-
6	$-2.6\cdot10^{-5}$	$-2.47\cdot10^{-5}$	$-2.35\cdot10^{-5}$	$-2.24\cdot10^{-5}$	-
7	$-2.55\cdot10^{-5}$	$-2.43\cdot10^{-5}$	$-2.31\cdot10^{-5}$	$-2.2\cdot10^{-5}$	
8	$-2.5\cdot10^{-5}$	$-2.38\cdot10^{-5}$	$-2.26\cdot10^{-5}$	$-2.16\cdot10^{-5}$	-
9	$-2.44\cdot10^{-5}$	$-2.32\cdot10^{-5}$	$-2.21\cdot10^{-5}$	$-2.11\cdot10^{-5}$	-
10	$-2.37\cdot10^{-5}$	$-2.26\cdot10^{-5}$	$-2.16\cdot10^{-5}$	$-2.06\cdot10^{-5}$	-
11	$-2.3\cdot10^{-5}$	$-2.2\cdot10^{-5}$	$-2.1\cdot10^{-5}$	$-2.01\cdot10^{-5}$	-
12	$-2.23\cdot10^{-5}$	$-2.13\cdot10^{-5}$	$-2.04\cdot10^{-5}$	$-1.95\cdot10^{-5}$	-
13	$-2.15\cdot10^{-5}$	$-2.06\cdot10^{-5}$	$-1.97\cdot10^{-5}$	$-1.89\cdot10^{-5}$	-
14	$-2.07\cdot10^{-5}$	$-1.98\cdot10^{-5}$	$-1.9\cdot10^{-5}$	$-1.82\cdot10^{-5}$	-
15	$-1.99\cdot10^{-5}$	$-1.91\cdot10^{-5}$	$-1.83\cdot10^{-5}$	$-1.76\cdot10^{-5}$	

BYrounded_ext2 =

	0	1	2	3	4
0	0	0	0	0	0
1	0	$1.415\cdot10^{-3}$	$2.373\cdot10^{-3}$	$2.719\cdot10^{-3}$	$2.591\cdot10^{-3}$
2	0	$3.541\cdot10^{-3}$	$5.711\cdot10^{-3}$	$6.222\cdot10^{-3}$	$5.65\cdot10^{-3}$
3	0	$7.881\cdot10^{-3}$	0.012	0.012	$9.655\cdot10^{-3}$
4	0	0.019	0.025	0.02	0.015
5	0	0.057	0.055	0.034	0.021
6	0	0.228	0.111	0.045	0.024
7	0	0.11	0.082	0.041	0.023
8	0	0.032	0.037	0.027	0.018
9	0	0.012	0.017	0.016	0.013
10	0	$5.735\cdot10^{-3}$	$9.095\cdot10^{-3}$	$9.777\cdot10^{-3}$	$8.896\cdot10^{-3}$
11	0	$3.041\cdot10^{-3}$	$5.169\cdot10^{-3}$	$6.083\cdot10^{-3}$	$6.055\cdot10^{-3}$
12	0	$1.778\cdot10^{-3}$	$3.155\cdot10^{-3}$	$3.941\cdot10^{-3}$	$4.181\cdot10^{-3}$
13	0	$1.117\cdot10^{-3}$	$2.04\cdot10^{-3}$	$2.653\cdot10^{-3}$	$2.948\cdot10^{-3}$
14	0	[illegible]	[illegible]	[illegible]	[illegible]

14	0	$7.422 \cdot 10^{-4}$	$1.381 \cdot 10^{-3}$	$1.849 \cdot 10^{-3}$	$2.126 \cdot 10^{-3}$
15	0	$5.149 \cdot 10^{-4}$	$9.716 \cdot 10^{-4}$	$1.328 \cdot 10^{-3}$	...

9	80
[illegible]	[illegible]

$51 \cdot 10^{-4}$	$-5.32 \cdot 10^{-4}$
$.5 \cdot 10^{-4}$	$-5.32 \cdot 10^{-4}$
$.5 \cdot 10^{-4}$	$-5.31 \cdot 10^{-4}$
$48 \cdot 10^{-4}$	$-5.3 \cdot 10^{-4}$
$46 \cdot 10^{-4}$	$-5.27 \cdot 10^{-4}$
$43 \cdot 10^{-4}$	$-5.25 \cdot 10^{-4}$
$.4 \cdot 10^{-4}$	$-5.22 \cdot 10^{-4}$
$36 \cdot 10^{-4}$	$-5.18 \cdot 10^{-4}$
$31 \cdot 10^{-4}$	$-5.14 \cdot 10^{-4}$
$26 \cdot 10^{-4}$	$-5.09 \cdot 10^{-4}$
$21 \cdot 10^{-4}$	$-5.04 \cdot 10^{-4}$
$14 \cdot 10^{-4}$	$-4.98 \cdot 10^{-4}$
$08 \cdot 10^{-4}$	$-4.91 \cdot 10^{-4}$
$01 \cdot 10^{-4}$	$-4.85 \cdot 10^{-4}$
$93 \cdot 10^{-4}$	$-4.77 \cdot 10^{-4}$
$35 \cdot 10^{-4}$	...

4
$3.39 \cdot 10^{-4}$
$6.8 \cdot 10^{-4}$
$1.039 \cdot 10^{-3}$
$1.429 \cdot 10^{-3}$
$1.86 \cdot 10^{-3}$
$2.348 \cdot 10^{-3}$
$2.91 \cdot 10^{-3}$
$3.57 \cdot 10^{-3}$
$4.358 \cdot 10^{-3}$
$5.312 \cdot 10^{-3}$
$6.486 \cdot 10^{-3}$
$7.95 \cdot 10^{-3}$
$9.802 \cdot 10^{-3}$
0.012
0.015
...

60
$2.23 \cdot 10^{-5}$
$2.22 \cdot 10^{-5}$
$2.21 \cdot 10^{-5}$
$-2.2 \cdot 10^{-5}$
$2.18 \cdot 10^{-5}$
$2.16 \cdot 10^{-5}$
$2.13 \cdot 10^{-5}$
$-2.1 \cdot 10^{-5}$
$2.06 \cdot 10^{-5}$
$2.02 \cdot 10^{-5}$
$1.97 \cdot 10^{-5}$
$1.92 \cdot 10^{-5}$
$1.87 \cdot 10^{-5}$
$1.81 \cdot 10^{-5}$
$1.75 \cdot 10^{-5}$
...