# Trajectories for Rutherford Scattering

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Here is the algorithm for computing the alpha particle trajectories for the Plum Pudding, Bohr, deBroglie, and Schrodinger models in the "Models of the Hydrogen Atom" simulation. The only difference between the models is the value of the constant D, discussed below.

**Variables and Constants:**

$x$,$y$ = coordinates in Cartesian coordinates
$r$,$\phi$ = coordinates in Polar coordinates
$t$ = time
$v$ = particle speed
$v_o$ = initial particle speed
$\Delta t$ = size of time step
$L$ = width of box.
$b = \frac{1}{2}\left(x_o + \sqrt{-2D\sqrt{x_o^2+y_o^2} - 2Dy_o + x_o^2}\right)$ = horizontal distance at $y = -\infty$
$R$ = Radius of red blob in Plum Pudding model (can be approximate).
$D = 2ke^2/(mv_0^2/2)$ = a constant with units of length.
For Bohr/deBroglie/Schrodinger, $D = D_B = L/16$ (We'll probably need to tweak this number a bit.)
For Plum Pudding, $D = D_P = \begin{cases} D_B x_o^2/R^2 & \text{if } x_o \leq R, \\ D_B & \text{if } x_o \geq R. \end{cases}$
Note that in Bohr/deBroglie/Schrodinger, $D$ is the same for all particles, but in Plum Pudding, $D$ depends on the initial position of the particle in the box.

**Conversion from Cartesian coordinates to Polar coordinates:**

$r = \sqrt{x^2+y^2}$ $\qquad$ $\phi = arctan(-x/y)$

**Conversion from Polar coordinates to Cartesian coordinates:**

$x = r\sin(\phi)$ $\qquad$ $y = -r\cos(\phi)$

**Initial Conditions:**

$x = b$ $\qquad$ $y = -L/2$ $\qquad$ $v = v_o$
Convert $x$ and $y$ to $r$ and $\phi$ to input into loop:

**Loop:**

1. $\phi_{new} = \phi + \frac{b^2 v_\Delta t}{r\sqrt{b^4 + r^2(b\cos\phi - \frac{D}{2}\sin\phi)^2}}$
2. $r_{new} = \frac{b^2}{b\sin(\phi_{new}) + \frac{D}{2}(\cos(\phi_{new})-1)}$
3. $v_{new} = v_o\sqrt{1 - D/r_{new}}$
4. Convert $r_{new}$ and $\phi_{new}$ to $x_{new}$ and $y_{new}$ and move alpha particle to new coordinates.
5. $r_{new}$, $\phi_{new}$, $v_{new} \rightarrow r$, $\phi$, $v$ for next iteration of loop.

Note: You will run into divide by zero problems when b=0. I would advise just making sure that x0 is never exactly zero..